INRIA

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Message Delay in MANET

Robin Groenevelt — Philippe Nain — Ger Koole

N° 5372

November 2004

Thème COM

Rapport de recherche

ISSN 0249-6399 ISRN INRIA/RR--5372--FR+ENG

Message Delay in MANET

Robin Groenevelt* , Philippe Nain* , Ger Koole†

Thème COM — Systèmes communicants
Projet MAESTRO

Rapport de recherche n° 5372 — November 2004 — 25 pages

Abstract: A stochastic model is introduced that accurately models the message delay in a Mobile Ad Hoc Network (MANET) where nodes can relay messages. The model has only two input parameters: the number of nodes and the intensity of a finite number of homogeneous and independent Poisson processes modeling instances when any pair of nodes come within transmission range of one another. Closed-form expressions are obtained for the Laplace-Stieltjes transform of the message delay, defined as the time needed to transfer a message between a source and a destination. From this result, we derive the expected message delay in closed-form as well as its asymptotic expansion for large networks. The probability distribution of the number of copies of the message at the time the message is delivered is also computed. These calculations are carried out for two relay protocols, the two-hop relay and the unrestricted relay protocols. Despite its simplicity, the model is able to accurately predict the performance of both relay protocols for a number of mobility models (Random Waypoint, Random Direction and Random Walker Mobility Models), as shown by simulations.

Key-words: Mobile ad hoc network; Routing protocol; Markov chain; Mobility model.

* Email: {robin.groenevelt}{philippe.nain}@sophia.inria.fr
† Email: {koole@few.vu.nl}

Unité de recherche INRIA Sophia Antipolis
2004, route des Lucioles, BP 93, 06902 Sophia Antipolis Cedex (France)
Téléphone : +33 4 92 38 77 77 —Télécopie : +33 4 92 38 77 65

Délai dans les Réseaux Ad Hoc Mobiles

Résumé : Nous proposons un modèle stochastique pour le calcul du temps de transfert d'un message dans un réseau ad hoc mobile, dans lequel les nœuds mobiles (ou simplement mobiles) servent de nœuds relais. Le modèle a deux paramètres d'entrée: le nombre de mobiles et l'intensité d'un nombre fini de processus de Poisson homogènes et indépendants. Ces processus de Poisson représentent les instants auxquels deux mobiles arbitraires peuvent communiquer. Nous calculons la transformée de Laplace-Stieltjes du temps de transfert (ou délai) d'un message, défini comme le temps nécessaire pour transmettre un message d'une source à une destination. De ce résultat nous déduisons le temps moyen de transfert d'un message, ainsi que son comportement asymptotique pour de grands réseaux. Nous déterminons également la distribution du nombre de copies d'un message dans le réseau à l'instant où le message atteint sa destination. Ces calculs sont effectués pour deux types de protocoles de routage qui utilisent les mobiles comme relais: le *two-hop relay protocol*, où seule la source peut utiliser les mobiles comme relais, et le *unrestricted relay protocol*, où tous les mobiles peuvent utiliser les autres mobiles comme relais. En dépit de sa simplicité, le modèle est capable de prédire les performances (délai, nombre de copies) de ces deux protocoles pour trois modèles différents de mobilité (*Random Waypoint*, *Random Direction*, *Random Walker*), comme le montrent des résultats de simulations.

Mots-clés : Réseau ad hoc mobile; Protocole de routage; Chaîne de Markov; Modèles de mobilité.

1 Introduction

In Mobile Ad Hoc Networks (MANET) a mobile node (or simply a node) can only send data to another node if both nodes are within transmission range of one another or *in contact*. Two nodes are within transmission range of one another if the distance between them does not exceed R.

The fact that two nodes are in contact is of course not enough to ensure the success of a transmission, since many phenomena may occur during the transmission and cause it to fail (interferences due to other transmissions, physical obstacles, power problems, etc.). Message relaying is a technique that facilitates the delivery of a message by using intermediary nodes to forward the message.

Routing protocols using relay nodes [9, 10, 17] have recently been proposed that increase the message delivery ratio in mobile ad hoc networks. These protocols operate on a *store-carry-forward* mode to take advantage of node mobility to improve node connectivity, and ultimately the message throughput. When information is available (node movement, node position, etc.) these protocols may use it in a static [17] or in a dynamic [9] way. The concept of relay nodes can also be used in the case when no information on the nodes is available [10].

Evaluating the performance of relay protocols (message delivery ratio, message latency, throughput, etc.) is a difficult task due to the inherent complexity of mobile ad hoc networks, particularly the random nature of both the movement of the nodes and of the demand (traffic). The performance of mobile ad hoc networks are in general studied via lengthy and complex simulations, for a limited number of mobility models, including the *Random Waypoint Mobility Model* [4, 8] or the *Random Direction Mobility Model* [2, 7].

In this paper we introduce a simple stochastic model to evaluate the performance of relay protocols for mobile ad hoc networks. The model has only two input parameters: the number of nodes in the network and the intensity (λ) of some identical and independent Poisson processes. In particular, the model does not require knowledge of the stationary distribution of the location of the nodes as input.

These processes model instances, called *meeting times*, at which any pair of nodes come within transmission range of one another. Transmissions between two nodes can only take place at meeting times and are assumed to be instantaneous. The latter assumption models the situation where the transmission time of a message is very small with respect to the time needed for two nodes to meet. Therefore, the random nature of a MANET is captured in our model through a finite number of these independent and homogeneous Poisson processes.

The selection of the intensity λ will be discussed in Sections 3 and 4.

The model is used to characterize the message delay between two arbitrary nodes – hereafter called the *source node* and the *destination node* – for two relay protocols and for three mobility models.

The two relay protocols are the *two-hop relay* protocol and the *unrestricted relay* protocol.

In the two-hop relay protocol the source node may forward the message to all the nodes it meets along its route, including of course the destination node. Any node which has received the message from the source node may only forward it to the destination node.

In the unrestricted relay protocol the source node may forward a message to all the nodes it meets (as in the two-hop relay protocol), but in this protocol any node that carries the message may in turn forward the message to all the nodes *it* encounters, along *its* trajectory.

The three mobility models that we will consider in this paper are the Random Waypoint Mobility Model and the Random Direction Mobility Model (both mentioned above), and the *Two-dimensional Random Walker Mobility Model*. All three models and their mathematical properties will be carefully described in Section 3.1.

The characterization of the message delay in MANET has already received some attention. In [14] it is shown that, under the two-hop relay protocol, the expected message delay is of the order $T_p(n)n$ for the Random Waypoint Mobility Model on a sphere (where n is the number of nodes per unit area and $T_p(n)$ is the transmission time of a message). It is shown in the same paper that the expected message delay is of the order $T_p(n)\log^2(n)$ when nodes execute independent Brownian motions on a sphere. In [6] the expected message delay is computed for a unidimensional network topology, where the nodes move in adjacent segments according to independent and reflected Brownian motions.

The paper is organized as follows: the stochastic model is introduced in Section 2.1, then we compute in Section 2.2 the Laplace-Stieltjes Transform (LST) of the message delay (Proposition 2.1). In this proposition, we also obtain the distribution of the number of copies of the message at the time the message is delivered to the destination node. In Proposition 2.2 we calculate the expected message delay in closed-form. From this, we derive an asymptotic expansion of the expected message delay for a large number of nodes. These calculations are done for the two relay protocols.

In Section 3, the expected message delay and the distribution of the number of copies of the message found in Section 2 are compared to results obtained by simulations. The simulations have been carried out for each of the six combinations of the two relay protocols and the three mobility models. The simulation results are very close to the analytical results. We observed discrepancies only when the node transmission range is large with respect to the size of the area in which the nodes move. We also show that the parameter λ of the stochastic model behaves as a quadratic function of the transmission range.

The model assumptions have been validated in Section 3 in the absence of interferences (a situation that will typically occur when the communication radius of the nodes is small with respect to the area in which the nodes move, and the node density is also small). One way to incorporate interferences into our model is to thin the meeting time sequences: with some (independent) probability p (resp. $1-p$) a transmission occurring at a meeting time will be a success (resp. failure). Due to the fact that a thinned Poisson process is again a Poisson process, it is enough to replace λ by λp, with p the probability that a communication fails due to interferences. On-going research is devoted to this issue.

On the other hand, we may also argue that the communication radius of the nodes must be small enough so that interferences remain at an acceptable level. It has been shown in [15, Lemma 1] that the transmission range of the nodes should be of the order $1/\sqrt{N}$ for the two-hop relay protocol, in order to maintain a constant capacity per node (with N the number of nodes per unit area). In Section 4 we show how our model can be used to compute the expected message delays for large networks and for the two relay protocols considered in this paper, when the transmission range is of the order of $1/\sqrt{N}$.

A word on the notation: given a function $g(N)$, we write $f(N) = \mathcal{O}(g(N))$ if $|f(N)/g(N)|$ is bounded from above as $N \to \infty$ and $f(N) = o(g(N))$ if $f(N)/g(N) \to 0$ as $N \to \infty$.

2 The Stochastic Model

We consider a network with $N+1$ identical mobile nodes. There is a *single message* to be delivered by a source node to a destination node. Intermediary nodes can be used as relay nodes. The goal is to determine the distribution of the message delay and the distribution of the number of copies of the message at the time the message is delivered to the destination node.

We first introduce the model (Section 2.1); then we use it in Section 2.2 to evaluate the performance (message delay, number of copies) of the two-hop relay and the unrestricted relay protocols.

2.1 Definition of the Model

An analytical model that would carefully take into account the main features of a MANET (transmission range, mobility pattern, interferences, fading, etc.) would be mathematically intractable. Instead, we propose a model where the impact of these features are captured through a single parameter (the parameter λ, see below).

Let $0 \le t_{i,j}(1) < t_{i,j}(2) < \cdots$ be the successive *meeting times* between nodes i and j ($i \neq j$). Define $\tau_{i,j}(n) := t_{i,j}(n+1) - t_{i,j}(n)$, the n-th *inter-meeting time* between nodes i and j.

Transmissions between two nodes may only take place at meeting times and are assumed to be *instantaneous*. The latter assumption covers the situation where the transmission time of a message between two nodes is negligible with respect to the node inter-meeting times.

We assume that *if* a transmission takes place between node i and node j (at some meeting time $t_{i,j}(n)$) then it will be *successful*. Assume that node i carries the message just before time $t_{i,j}(n)$. Under the two-hop relay protocol node i will transmit (a copy of) the message to node j at time $t_{i,j}(n)$ if i is the source node or if j is the destination node. Under the unrestricted relay protocol node i will always transmit the message to node j at time $t_{i,j}(n)$ regardless of the identity of node j.

Throughout the paper the following assumption will hold for each relay protocol:

(A) the processes $\{t_{i,j}(n), n \ge 1\}$, $1 \le i,j \le N+1$, $i \neq j$, are mutually independent and homogeneous Poisson processes with rate $\lambda > 0$. Equivalently stated, the random variables (rvs) $\{\tau_{i,j}(n)\}_{i,j,n}$ are mutually independent and exponentially distributed with mean $1/\lambda$.

We introduce:

- T_2 (resp. T_U), the message delay under the two-hop (resp. unrestricted) relay protocol, defined as the time needed to send the message (or a copy of the message) from the source to the destination;
- $N_2 \in \{1, 2, \ldots, N\}$ (resp $N_U \in \{1, 2, \ldots, N\}$), the number of copies of the message in the network (including the original message but excluding the message at the destination node) at the time the message is delivered to the destination node.

For $\theta \geq 0$ let

$$T_2^\star(\theta) := E[e^{-\theta T_2}], \quad T_U^\star(\theta) := E[e^{-\theta T_U}]$$

be the LST of T_2 and T_U, respectively.

2.2 Performance of relay protocols

Proposition 2.1 gives, for each relay protocol, the LST of the message delay and the distribution of the number of copies.

Proposition 2.1 (LST of message delay).
Under the two-hop relay protocol

$$T_2^\star(\theta) = \sum_{i=1}^{N} i \, \frac{(N-1)!}{(N-i)!} \left(\frac{\lambda}{\lambda N + \theta} \right)^i \tag{1}$$

and

$$P(N_2 = i) = \frac{i}{N^i} \, \frac{(N-1)!}{(N-i)!}, \quad i = 1, \ldots, N. \tag{2}$$

Under the unrestricted relay protocol

$$T_U^\star(\theta) = \frac{1}{N} \sum_{i=1}^{N} \prod_{j=1}^{i} \frac{\lambda j (N+1-j)}{\lambda j (N+1-j) + \theta} \tag{3}$$

and

$$P(N_U = i) = \frac{1}{N}, \quad i = 1, \ldots, N, \tag{4}$$

that is, the number of copies is uniformly distributed over $\{1, \ldots, N\}$. $\diamond$

Proof. For both the two-hop and the unrestricted protocols the proof is based on modeling the number of copies in the network as an absorbing finite-state Markov chain. The transition rates of these Markov chains will differ for each protocol.

For each protocol the Markov chain takes its values in $\{1, 2, \ldots, N+1\}$. The Markov chain is in state $i = 1, 2, \ldots, N$ when there are i copies of the message in the network including the original message, and it is in state $N+1$ when the message has been delivered to the destination node. Note that states $1, 2, \ldots, N$ are transient states and $N+1$ is an absorbing state.

We provide a separate proof for (1)-(2) and (3)-(4).

Proof of (1) and (2):

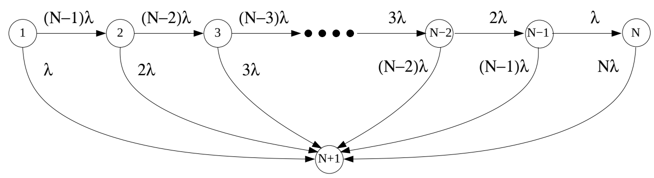

Figure 1: Two-hop relay: transition diagram of the Markov chain representing the number of copies

The transition diagram of the Markov chain corresponding to the two-hop relay protocol is given in Figure 1. Recall that under the two-hop relay protocol only the source node distributes copies of the message to nodes that come within its transmission range. Therefore, when there are i copies in the network, then either a new copy is sent to the $N-i$ nodes which do not have a copy yet, which occurs at the rate $\lambda(N-i)$ and triggers a transition from i to $i+1$, or one of these i copies reaches the destination node, which occurs at the rate λi and triggers a transition from i to $N+1$. This explains the transition diagram in Figure 1.

The transition from i from $i+1$ occurs with the probability $(N-i))\lambda/((N-i))\lambda+i\lambda) = 1-i/N$, and the transition from i to $N+1$ occurs with the complementary probability $i\lambda/((N-i))\lambda+i\lambda) = i/N$.

The sojourn time S_i in state $i = 1, 2, \ldots, N$ is exponentially distributed with intensity λN (the sum of transition rates out of state i). Moreover $S_1, \ldots, S_N$ are mutually independent random variables.

By conditioning on the state of the Markov chain just before its enters state $N+1$, or equivalently by conditioning on the number of copies N_2 just before the message hits its destination, we have

$$\begin{aligned} T_2^\star(\theta) =& \sum_{i=1}^{N} \mathbb{E}[e^{-\theta T_2} \mid N_2 = i] P(N_2 = i) \\ =& \sum_{i=1}^{N} \mathbb{E}[e^{-\theta \sum_{j=1}^{i} S_j} \mid N_2 = i] P(N_2 = i). \end{aligned} \tag{5}$$

As mentioned earlier, $1 - j/N$ (resp. j/N) is the probability of jumping from state j to state $j+1$ (resp. $N+1$). Therefore,

$$P(N_2 = i) = \frac{i}{N} \prod_{j=1}^{i-1} \left(1 - \frac{j}{N}\right) = \frac{i}{N^i} \frac{(N-1)!}{(N-i)!}, \tag{6}$$

which establishes (2).

When in state $j = 1, 2, \ldots, N$, the Markov chain can either enter state $j+1$ after a time $S_{j,1}$ that is exponentially distributed with intensity $(N+1-j)\lambda$, or enter state $N+1$ after a time $S_{j,2}$, independent of $S_{j,1}$, and exponentially distributed with intensity $j\lambda$. Observe that $S_j = \min\{S_{j,1}, S_{j,2}\}$. Moreover

$$\begin{aligned} P[S_{j,1} < x \mid S_{j,1} < S_{j,2}] =& P[S_{j,2} < x \mid S_{j,1} > S_{j,2}] \\ =& P(S_j < x) \\ =& 1 - \exp(-\lambda N) \end{aligned} \tag{7}$$

as a consequence of the exponential distribution. Therefore,

$$\begin{aligned} \mathbb{E}[e^{-\theta \sum_{j=1}^{i} S_j} \mid N_2 = i] =& \mathbb{E}[e^{-\theta(\sum_{j=1}^{i-1} S_{j,1} + S_{i,2})} \mid S_{j,1} < S_{j,2}, \\ & \ldots, S_{i-1,1} < S_{i-1,2}, S_{i,1} > S_{i,2}] \end{aligned} \tag{8}$$

>From (7), (8) and the fact that the rvs $\{S_{j,k}\}_{j=1,\ldots,N,k=1,2}$ are mutually independent, we readily find

$$\mathbb{E}[e^{-\theta \sum_{j=1}^{i} S_j} \mid N_2 = i] = \prod_{j=1}^{i} \mathbb{E}[e^{-\theta S_j}] = \left(\frac{\lambda N}{\lambda N + \theta}\right)^i. \tag{9}$$

Putting (5), (6) and (9) together yields

$$T_2^\star(\theta) = \sum_{i=1}^{N} i \frac{(N-1)!}{(N-i)!} \left(\frac{\lambda}{\lambda N + \theta}\right)^i,$$

which proves (1).

Proof of (3) and (4):

The transition diagram of the Markov chain associated with the unrestricted relay protocol is displayed in Figure 2. Under this protocol, each node which has a copy of the message is allowed to distribute it to a node which does not have a copy and which comes within its transmission range. Therefore, when there are i copies of the message in the network a new copy is created at the rate $\lambda i(N-i)$ (transition from i to $i+1$) and one of these i copies reaches the destination node at the rate λi (transition from i to $N+1$), as depicted on Figure 2.

The chain jumps from state i to state $i+1$ with probability $(N-i)/(N+1-i)$ and it jumps from state i to state $N+1$ with probability $1/(N+1-i)$. The sojourn time $\tilde{S}_i$ in

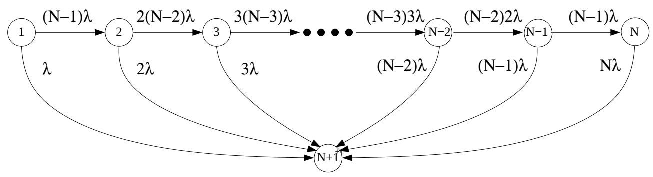

Figure 2: Unrestricted relay: transition diagram of the Markov chain representing the number of copies

state i is exponentially distributed with intensity $\lambda i(N+1-i)$ (obtained as the sum of the transition rates going out state i).

By conditioning on the number of copies N_U, we have

$$T_U^\star(\theta) = \sum_{i=1}^{N} \mathbb{E}[e^{-\theta \sum_{j=1}^{i} \tilde{S}_j} \mid N_U = i] P(N_U = i)$$

with

$$P(N_U = i) = \frac{1}{N+1-i} \prod_{j=1}^{i-1} \frac{N-j}{N+1-j} = \frac{1}{N},$$

which proves (4).

Similarly to (9) we have

$$\mathbb{E}[e^{-\theta \sum_{j=1}^{i} \tilde{S}_j} \mid N_U = i] = \prod_{j=1}^{i} \mathbb{E}[e^{-\theta \tilde{S}_j}] = \prod_{j=1}^{i} \frac{\lambda j(N+1-j)}{\lambda j(N+1-j)+\theta},$$

so that

$$T_U^\star(\theta) = \frac{1}{N} \sum_{i=1}^{N} \prod_{j=1}^{i} \frac{\lambda j(N+1-j)}{\lambda j(N+1-j)+\theta},$$

which proves (3). ∎

Proposition 2.2 gives the expected message delay for each relay protocol. This result shows that for each protocol the expected message delay is a linear function of the expected inter-meeting time $1/\lambda$.

Proposition 2.2 (Expected message delays). *Under the two-hop relay protocol, the expected message delay is given by*

$$\mathbb{E}[T_2] = \frac{1}{\lambda N} \sum_{i=1}^{N} \frac{i^2 (N-1)!}{(N-i)!\, N^i} \tag{10}$$

$$= \frac{1}{\lambda} \left(\sqrt{\frac{\pi}{2N}} + \mathcal{O}\left(\frac{1}{N}\right) \right). \tag{11}$$

Under the unrestricted relay protocol, the expected message delay is given by

$$\mathbb{E}[T_U] = \frac{1}{\lambda N} \sum_{i=1}^{N} \frac{1}{i} \tag{12}$$

$$= \frac{1}{\lambda N} \left(\log(N) + \gamma + \mathcal{O}\left(\frac{1}{N}\right) \right), \tag{13}$$

where $\gamma \approx 0.57721$ *is Euler's constant.*

Proof. Since $\mathbb{E}[T_2] = -\left.\frac{dT_2^\star(\theta)}{d\theta}\right|_{\theta=0}$, $\mathbb{E}[T_2]$ can be derived at once from (1). For sake of clarity the proof of (11) is forwarded to the appendix.

Similarly, we find by differentiating (3) w.r.t θ, and then by setting $\theta = 0$, that

$$\begin{aligned}
\mathbb{E}[T_U] &= \frac{1}{\lambda N} \sum_{i=1}^{N} \sum_{j=1}^{i} \frac{1}{j(N+1-j)} \\
&= \frac{1}{\lambda N(N+1)} \sum_{i=1}^{N} \sum_{j=1}^{i} \left(\frac{1}{j} + \frac{1}{N+1-j} \right) \\
&= \frac{1}{\lambda N(N+1)} \sum_{j=1}^{N} \sum_{i=j}^{N} \left(\frac{1}{j} + \frac{1}{N+1-j} \right) \\
&= \frac{1}{\lambda N(N+1)} \sum_{j=1}^{N} \left(\frac{1}{j} + \frac{1}{N+1-j} \right) (N+1-j) \\
&= \frac{1}{\lambda N} \sum_{j=1}^{N} \frac{1}{j},
\end{aligned}$$

which is (12). This last summation is known as the harmonic numbers. Its asymptotic expansion is [13, p. 186]

$$\sum_{j=1}^{N} \frac{1}{j} = \log(N) + \gamma + \mathcal{O}\left(\frac{1}{N}\right),$$

where γ is Euler's constant. This gives (13) and concludes the proof. ∎

The next result gives the expected number of copies of the message at the time the message is delivered.

Corollary 2.1 (Expected number of copies). *The expected number of copies under the two-hop relay protocol is given by (cf. (2))*

$$\mathbb{E}[N_2] = \frac{1}{N} \sum_{i=1}^{N} \frac{i^2}{N^i} \frac{N!}{(N-i)!}. \tag{14}$$

Hence (cf. (10))

$$\mathbb{E}[N_2] = \lambda N \, \mathbb{E}[T_2],$$

so that

$$\mathbb{E}[N_2] = \sqrt{\frac{\pi N}{2}} + \mathcal{O}(1). \tag{15}$$

The expected number of copies under the unrestricted relay protocol is (cf. (4))

$$\mathbb{E}[N_U] = \frac{N+1}{2}.$$ ◇

The relative performance of the two-hop relay and unrestricted relay protocols can be captured through the ratios $\mathbb{E}[T_U]/\mathbb{E}[T_2]$ and $\mathbb{E}[N_U]/\mathbb{E}[N_2]$ given by (cf. Proposition 2.2)

$$\frac{\mathbb{E}[T_U]}{\mathbb{E}[T_2]} = \frac{N \sum_{i=1}^{N} \frac{1}{i}}{\sum_{i=1}^{N} \frac{i^2}{N^i} \frac{N!}{(N-i)!}}$$

and (cf. Corollary 2.1)

$$\frac{\mathbb{E}[N_U]}{\mathbb{E}[N_2]} = \frac{N(N+1)}{2 \sum_{i=1}^{N} \frac{i^2}{N^i} \frac{N!}{(N-i)!}},$$

respectively. Note that both ratios are independent of λ.

By using the asymptotic expansions (11), (13) and (15), we see that

$$\frac{\mathbb{E}[T_U]}{\mathbb{E}[T_2]} \approx \frac{\log(N)}{\sqrt{N}} \sqrt{\frac{2}{\pi}}$$

and

$$\frac{\mathbb{E}[N_U]}{\mathbb{E}[N_2]} \approx \sqrt{\frac{N}{2\pi}},$$

for large N. In other words, for large N the expected message delay under the unrestricted relay protocol is approximately $\log(N)/\sqrt{\pi N/2}$ times smaller than under the two-hop relay protocol, while the expected number of copies is approximately $\sqrt{N/2\pi}$ times larger.

For instance, for $N = 10^3$ then $\mathbb{E}[T_U]/\mathbb{E}[T_2] \approx 0.17$ and $\mathbb{E}[N_U]/\mathbb{E}[N_2] \approx 12.6$.

3 Applications

This section is devoted to the application of the results in Section 2 to three different mobility models. It is structured as follows: the mobility models are presented in Section 3.1 and the simulation setting for each mobility model is introduced in Section 3.2. Through both intuitive reasoning and simulations it is shown in Section 3.3 that assumption (A) is reasonable when the transmission range is not too large relative to the surface area. Based on this observation, estimates are obtained for the meeting rate λ, for each mobility model and for various transmission ranges. With the help of these estimates and Proposition 2.2, the expected message delays predicted by the analytical model are computed for each mobility pattern, for both the two-hop relay and for the unrestricted relay protocols, and are compared to simulation results.

The accuracy of the model is demonstrated in Sections 3.4 and 3.5, where the expected message delay and the distribution of the number of copies obtained by the model are compared to simulation results.

3.1 Mobility Models

Although the results in Section 2 hold regardless of the dimension of the space in which the nodes move, in the following we shall only apply them to three standard *two-dimensional* mobility models: the Random Waypoint Mobility Model (Section 3.1.1), the Random Direction Mobility Model (Section 3.1.2), and the Two-Dimensional Random Walker Mobility Model (Section 3.1.3).

3.1.1 Random Waypoint Mobility Model

The Random Waypoint Mobility Model [8, 4] is commonly used in the the simulation of mobile ad hoc networks. In the Random Waypoint Mobility Model each node is assigned an initial location in a given area (typically a square) and travels at a constant speed S to a destination chosen uniformly in this area. The speed S is chosen uniformly in (v_{min}, v_{max}), independently of the initial location and destination. After reaching the destination, the node may pause for a random amount of time after which a new destination and a new speed are chosen, independently of all previous destinations, speeds, and pause times. The stationary distributions of location and speed in the Random Waypoint Mobility Model differ significantly from the uniform distribution. In particular, it has been observed that the stationary distribution of the location of a node is more concentrated near the center of the region in which the nodes move [3]. Also, v_{min} needs to be strictly positive to ensure that the average speed over time does not go to zero [16]..

3.1.2 Random Direction Mobility Model

In the Random Direction Mobility Model [2, 7] each node is assigned an initial direction θ, speed $S \in [v_{\min}, v_{\max}]$ and a finite travel time τ. The node then travels in the direction θ

for a duration τ and at speed S. When the node travel time has expired a new direction, speed and travel time are chosen at random independently of all previous directions, speeds and travel times. When a node reaches a boundary it is either reflected [1, 2] or the area wraps around so that the node reappears on the other side [2].

The stationary distributions of the location and direction have been shown to be uniform [1, 11] for arbitrary direction, speed and travel time distributions, irrespective of the boundaries being reflecting or wrap around. This is in contrast with the Random Waypoint Mobility Model where nodes are more likely to be concentrated near the center of the area. Another difference is that the minimum speed v_{min} does not have to be strictly positive. The speed can be equal to zero since the node maintains a certain speed only for a limited amount of time.

3.1.3 Two-Dimensional Random Walker Model

In the two-dimensional Random Walker Mobility Model each node moves as a random walker on a two-dimensional square lattice. The time is discrete and at each time step each node has a probability of $1/4$ of hopping to a position above, below, to the left, or to the right of its current position. If the node is positioned on a boundary, then instead of hopping off the lattice it hops back to the same state. This movement can be seen as someone wandering at a constant speed from intersection to intersection through a city, where all of the streets are equally spaced and perpendicular to each other (Manhattan network). The stationary distribution of the location of a two-dimensional random walker on a square lattice is uniform over the area. This properties is a consequence of the fact that a two-dimensional random walker can be constructed from two independent one-dimensional random walkers, and that the stationary location of a symmetric random walk in one dimension is uniform (take $n \to \infty$ in Formula (3.15) in [5, p. 357] to obtain the stationary distribution and then set $p = q = \frac{1}{2}$).

3.2 Simulation Setting

The numerical results presented hereafter are based on simulation programs in which mobile nodes move in a square of size 4km$\times$4km, without pausing.

As mentioned in [12] there are several pitfalls to avoid when simulating the Random Waypoint Mobility Model. In this work we have used the implementation of the Random Waypoint Mobility Model proposed in [12] (without pausing) which consists of sampling the initial speeds and locations from their stationary distributions. Then, subsequent speeds and locations are sampled from the uniform distribution.

Since the stationary distribution of the location of a node is uniform in both the Random Direction Mobility Model and the Random Walker Mobility Model, their implementation does not pose any difficulty.

For the Random Waypoint Mobility Model and the Random Direction Mobility Model, a speed (in km/h) was chosen uniformly in $[v_{min}, v_{max}] = [4, 10]$.

In the Random Direction Mobility Model, a node moves in a direction that is uniformly distributed in $[0, 2\pi)$, for an exponentially distributed amount of time (expressed in hours) with mean $1/4$ and at a speed that is uniformly distributed in $[4, 10]$, before the node chooses a new direction, travel time and speed.

For the Random Walker Mobility Model we assume that the streets are 80 meters apart and that the random walkers move at the speed of one block per minute (this results in $51^2 = 2601$ states and a constant speed of 4.8 km/h).

We assume that there are no inferences and that the transmission of a message between two nodes (in contact) is instantaneous. The former assumption typically models a situation where the transmission radius is small with respect to the size of the area. The latter assumption typically models a situation where the message transmission time between two nodes is negligible with respect to the node inter-meeting times.

In order to apply the results in Section 2 we need, for each mobility model, to check the validity of assumption (A) and to identify the parameter λ of the exponential inter-meeting time distribution.

3.3 Validation of the Poisson meeting times

For each mobility model and for various communication radii, we have simulated the movement of two nodes and have estimated the distribution (Section 3.3.1) and the auto-covariance function (Section 3.3.2) of the inter-meeting times between these two nodes. The results, based on 100,000 observations and which are reported below, show that the Poisson assumption for the meeting time sequences is valid for all three mobility models and for a large range of communication radii.

3.3.1 Inter-Meeting Time Distribution

Figure 3 displays, on a log-scale for the y axis, the inverse Cumulative Distribution Function (CDF) of the inter-meeting time between two nodes for each mobility model and for three different communication radii ($R = 50m$, $100m$, $250m$). For each mobility model and for each communication radius we have also plotted the inverse CDF of an exponential distribution (i.e. a straight line on a log-scale for the y axis), whose intensity (i.e. slope) λ is the inverse of the expected inter-meeting time obtained across all simulations. For all nine cases that we have investigated (three mobility models and three different communication radii) we can observe an excellent agreement between the estimated CDF (solid line) and the exponential CDF (dashed line). Only for the Random Walker Mobility Model in the case where $R = 250m$ can we note some differences between the estimated CDF and the exponential CDF as the inter-meeting time increases.

The fact that, for each mobility model the CDF of the inter-meeting distribution is well-approximated by an exponential distribution, at least for small to moderate transmission radii (with respect to the size of the area) finds its roots in the various independence assumptions placed on each mobility model. Indeed, nodes move independently of each other and future directions and speeds (and therefore locations) of a node are independent of past

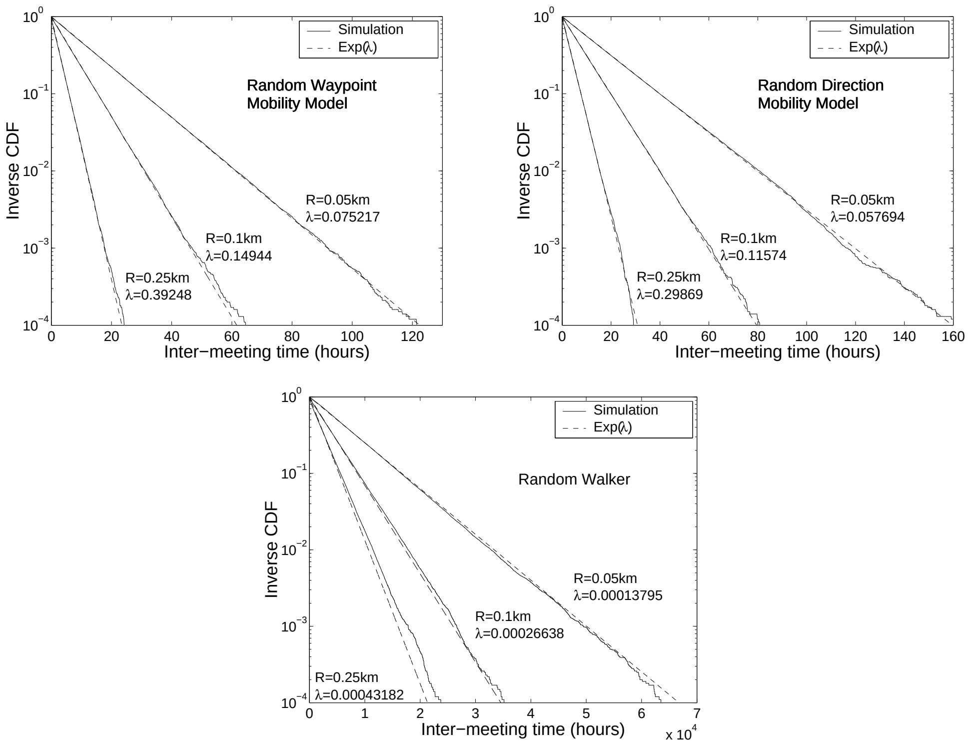

Figure 3: Inverse CDF of the inter-meeting time of two nodes for the the Random Waypoint Mobility Model, the Random Direction Mobility Model and the Random Walker Model.

directions and speeds of this node. If we pick two mobile nodes at random at some stationary time, then there is a probability q that they will meet (in the sense of being within transmission range of one another) before the next change of direction of either node. At the next change of direction, because of the independent assumptions recalled above, the process repeats itself and there is a probability q that these nodes will meet before the next change of direction. This yields a geometric distribution for the number of changes of direction before both nodes meet. The exponential distribution pops up because the number of changes of direction is "linearly" related to the time traveled before the nodes meet.

The fact that inter-meeting times are exponentially distributed has already been observed, via simulations, for the Brownian Mobility Model and the Random Waypoint Mobility Model on a sphere [14].

When estimating the intensity of the inter-meeting time λ, we have observed that this parameter exhibited a quadratic dependence on the communication radius R, as depicted

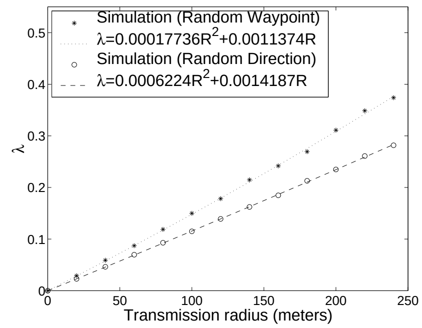

Figure 4: Relationship between the inter-meeting time intensity λ and the communication radius R

in Figure 4 for the Random Waypoint Mobility Model and the Random Direction Mobility Model (this observation also carries on to the Random Walker Mobility Model but due to the difference in scaling the results have not been included in the figure). Not surprisingly the quadratic relationship between λ and R depends on the mobility model, as shown in Figure 4.

Intuitively, this quadratic dependence follows from the fact that nodes can approach each other from any angle, which implies that inter-meeting times depend on the area defined by the node communication radius. This area is a disk of size πR^2, which gives the quadratic dependence of λ on R.

Also note that due to the absence of interferences λ does not depend on the relay protocol that is used.

3.3.2 Independence of Inter-Meeting Times

Let $\{\tau(n)\}_n$ be the inter-meeting times between two given nodes. To check the assumption that the rvs $\{\tau(n)\}_n$ are mutually independent rvs, we have used the following classical estimator for the auto-correlation function of $\{\tau(n)\}_n$

$$\rho_m(h) = \frac{\gamma_m(h)}{\gamma_0(h)}, \quad h \geq 0,$$

where

$$\gamma_m(h) := \frac{1}{m} \sum_{n=1}^{m-h} \left(\tau(n+h) - \hat{\tau}^{(m)}\right) \left(\tau(n) - \hat{\tau}^{(m)}\right)$$

is an estimator of the auto-covariance function, with $\hat{\tau}^{(m)} = (1/m) \sum_{n=1}^{m} \tau(n)$ the sample mean for m observations.

If the rvs $\{\tau(n)\}_n$ are mutually independent then their autocorrelation function is equal to zero for all $h \geq 1$.

The mapping $h \to \rho_m(h)$ corresponding to the Random Waypoint Mobility Model is plotted in Figure 5 for $m = 100,000$ and $R = 0.25km$. The autocorrelation functions corresponding to other values of R ($R = 0.05km$, $R = 0.1km$) and/or to the Random Direction Mobility Model and the Random Walker Mobility Model are not displayed since they are identical to the results in Figure 5. From these results we conclude that the assumption that the inter-meeting times between two nodes are mutually independent rvs is a reasonable assumption.

In conclusion, the results reported in Sections 3.3.1 and 3.3.2 validate the assumption that the meeting time process between two given nodes is a Poisson process for all three mobility models and for small to moderate communication radii (with respect to the size of the area in which the nodes move).

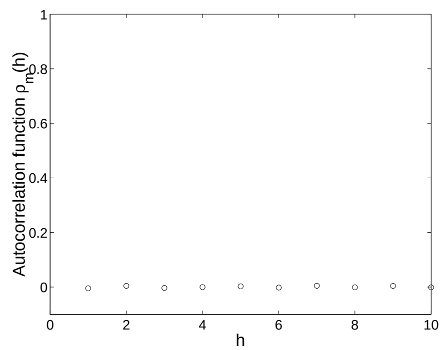

Figure 5: Autocorrelation function of inter-meeting times for the Random Waypoint Mobility Model with $R = 0.25km$

3.4 Expected Message Delay

For the three mobility models introduced in Section 3.1 and for three communication radii ($R = 0.05km$, $0.1km$, $0.25km$), Figures 6-7 display the expected message delays obtained both through simulations and by the analytical model as a function of the number of nodes. Results for the two-hop (resp. unrestricted) relay protocol are given in Figure 6 (resp. Figure 7).

These results demonstrate the ability of the analytical model to predict the expected message delay under both the two-hop relay protocol and the unrestricted relay protocol for different mobility patterns, across any number of nodes and communication radii.

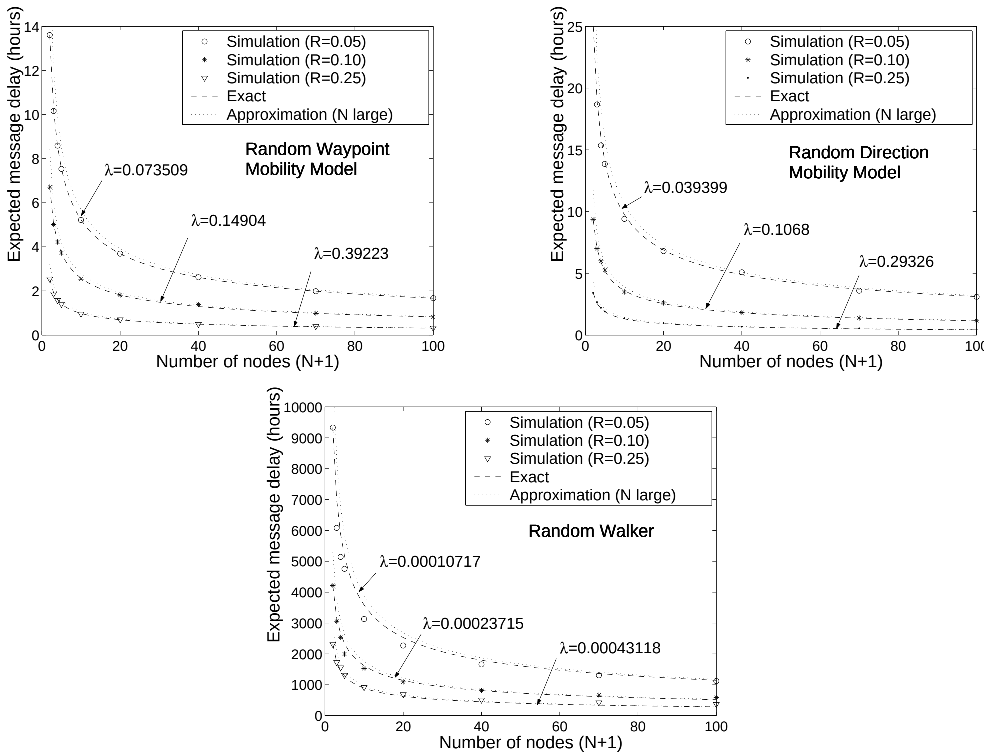

Figure 6: Expected message delay as a function of the number of nodes: the two-hop relay protocol

3.5 Distribution of Number of Copies

Figures 8-10 compare the distribution of the number of copies at message delivery time obtained through simulations (represented by bars in the figures) and by the analytical model (solid lines), under both relay protocols and for 40 nodes (i.e. $N = 39$).

Results for the two-hop relay protocol are displayed in Figures 8-9 for the Random Waypoint Mobility Model and the Random Walker Mobility Model, respectively (results for the Random Direction Mobility Models are identical to that of the Random Waypoint Mobility Model and have not been displayed). We observe that for all three mobility models the fit is quite good when $R = 50m$ and that it deteriorates as R increases (although the results are still acceptable for $R = 100m$ for the Random Waypoint Mobility Model and the Random Direction Mobility Model).

Results for the unrestricted relay protocol are reported in Figure 10. Recall that for this protocol the number of copies is uniformly distributed in the analytical model, namely,

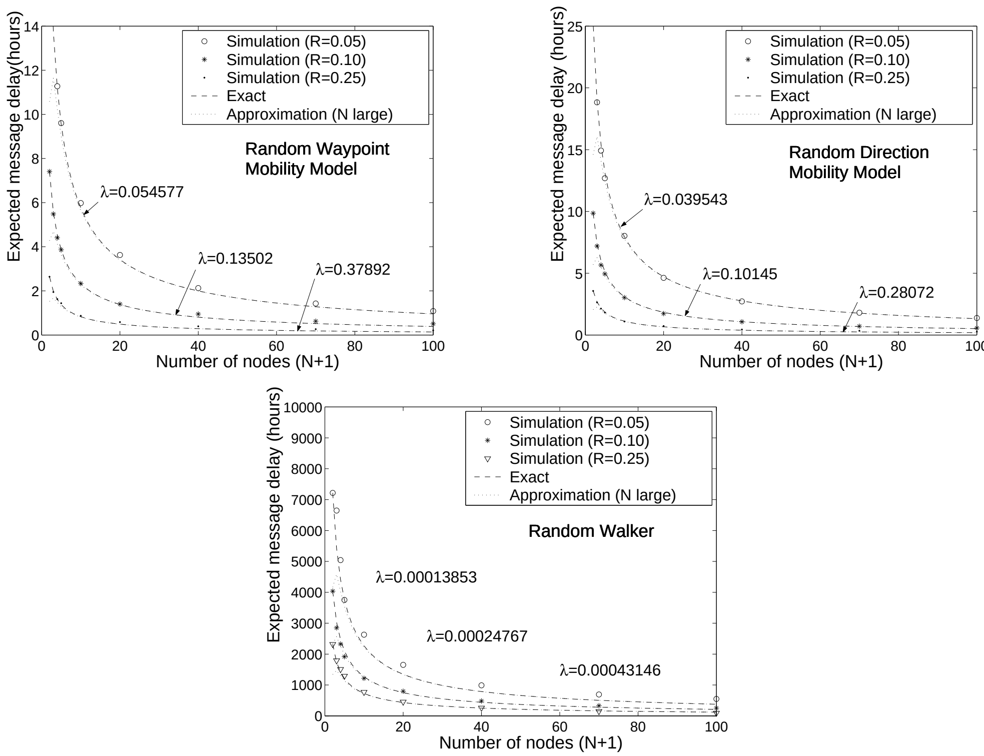

Figure 7: Expected message delay as a function of the number of nodes: the unrestricted relay protocol

$P(N_U = i) = 1/39 = 0.0256$ for all $i = 1, \ldots, 39$ (see Proposition 2.1). Results are displayed for each mobility model, each for a different transmission range. We can see that in all cases the distribution of the number of copies is very close to the uniform distribution.

These results give a good indication that our model, despite its genericness, is able to capture the main features of the interaction of the mobility models and the relay protocols.

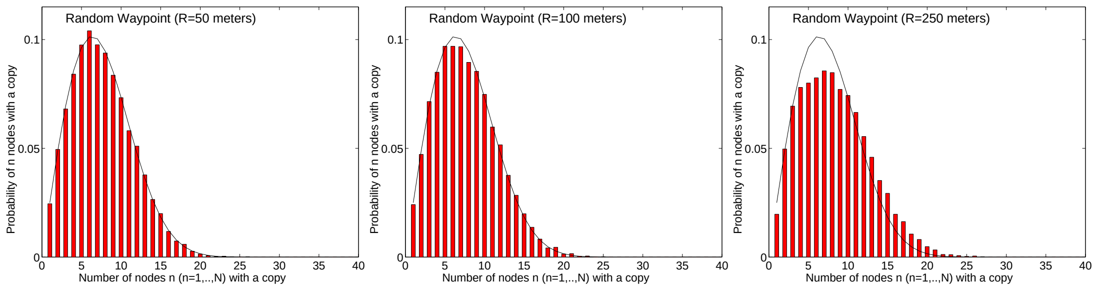

Figure 8: Distribution of the number of copies: the two-hop relay protocol under the Random Waypoint Mobility Model

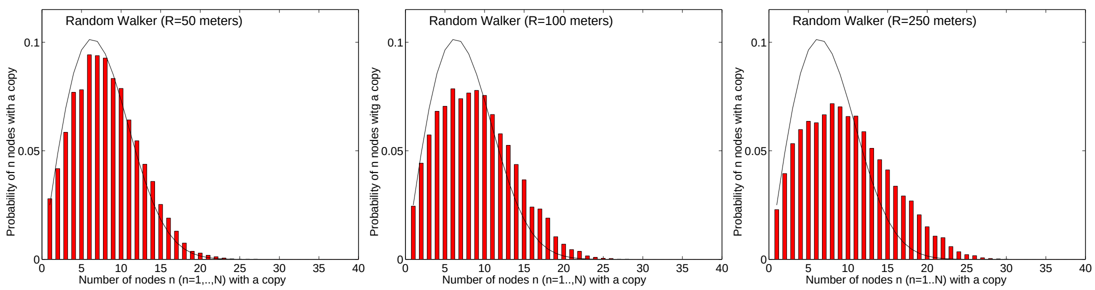

Figure 9: Distribution of the number of copies: the two-hop relay protocol under the Random Walker Mobility Model

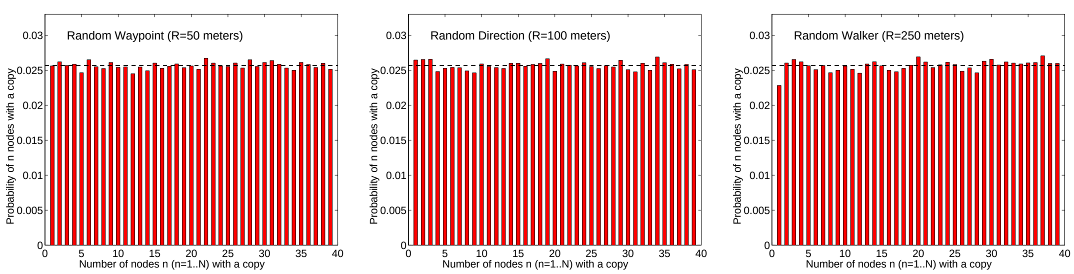

Figure 10: Distribution of the number of copies: the unrestricted relay protocol

4 Large networks

We have observed in Section 3 that, for the three mobility models considered in this paper, the inter-meeting time intensity (λ) is well approximated by a quadratic function of the transmission range R. This approximation is valid as long as R is not "too large" with respect to the size of the area in which the nodes move.

On the other hand, when the number of nodes increases R should decrease to prevent interferences from becoming excessive.

Putting these two observations together, yields

$$\lambda = \mathcal{O}(R^2(N)),$$

where $R(N)$, the transmission range for a network with N nodes, is a decreasing function of N.

Introducing this behavior of λ in Proposition 2.2 immediately gives the following:

Corollary 4.1. *For large N*

$$\mathbb{E}[T_2] = \mathcal{O}\left(\frac{1}{\sqrt{N}R^2(N)}\right),$$

$$\mathbb{E}[T_U] = \mathcal{O}\left(\frac{\log(N)}{NR^2(N)}\right). \qquad \diamond$$

If we choose $R(N) = \mathcal{O}(1/\sqrt{N})$ in order to keep interference at an acceptable level (it is shown in [15, Lemma 1] that $R(N) = \mathcal{O}(1/\sqrt{N})$ in order to achieve a constant capacity per node with the two-hop relay protocol), then

$$\mathbb{E}[T_2] = \mathcal{O}(\sqrt{N}) \tag{16}$$

and

$$\mathbb{E}[T_\infty] = \mathcal{O}(\log(N)),$$

by using Corollary 4.1.

Interestingly enough (16) was also obtained in [14, Corollary 4.1] for the two-hop relay protocol, through a different approach and for nodes moving according to the Random Waypoint Mobility Model on the surface of a sphere. In our case, however, this result is a direct consequence of the model, and is *independent* of the underlying mobility model and of (the dimension of) the area in which the nodes move.

Alternatively, one may want to find the function $R(N)$ so that the expected message delay is $\mathcal{O}(1)$ as the number of nodes becomes large. For the two-hop relay protocol this is achieved when $R(N) = \mathcal{O}(N^{1/4})$, whereas for the unrestricted relay protocol this is achieved when $R(N) = \mathcal{O}(\sqrt{\log(N)/N})$.

5 Conclusion

In this paper we have introduced a simple stochastic model to characterize the delay incurred by a message in a mobile ad hoc network. The model has two input parameters, the number of nodes, and the intensity of a finite number of independent and homogeneous Poisson processes. These processes model instances at which any pair of nodes come within transmission range of one another. The LST of the message delay, and the distribution of the number of copies of the message at the time the message is delivered to the destination node, have been derived for two relay protocols: the two-hop relay protocol and the unrestricted relay protocol. These analytical results have been compared to simulation results obtained for three different mobility models: the Random Waypoint, the Random Direction and the Random Walker Mobility Models. For small to moderate transmission radii (with respect to the size of the area), the analytical results are very close to the simulation results.

We believe that this generic model can be used to evaluate and compare the performance of different routing protocols for MANET for a wide range of mobility models.

Appendix

Lemma 1. *For large N*

$$\sum_{i=1}^{N} \frac{i^2 N!}{(N-i)!N^i} = N^{3/2}\sqrt{\frac{\pi}{2}} + \mathcal{O}(N). \tag{17}$$

Proof. Define $A(N) := \sum_{i=1}^{N} \frac{i^2 N!}{(N-i)!N^i}$.

If it were not for the presence of the factor i^2 in $A(N)$, then this quantity would be the Ramanujan Q-distribution [13, page 188], also known as the birthday function. This function often shows up in the analysis of algorithms.

The derivation of the approximation (17) follows that of the Ramanujan Q-distribution approximation [13, Proposition 4.8]. We now outline it.

Let $i_0 := \lfloor N^{3/5} \rfloor$. This implies that $i_0^2/N \to \infty$ as $N \to \infty$ and $i_0 = o(N^{2/3})$. We have

$$A(N) = \sum_{i=1}^{i_0} \frac{i^2 N!}{(N-i)!N^i} + B(N),$$

with $B(N) := \sum_{i=i_0+1}^{N} \frac{i^2 N!}{(N-i)!N^i}$.

$B(N)$ is an exponentially small function of N, in the sense that $B(N)$ is $\mathcal{O}(1/N^a)$ for any $a > 0$. The proof of this result goes as follows. It is shown in the proof of Proposition 4.8 in [13] that $C(N) := \sum_{i=i_0+1}^{N} \frac{N!}{(N-i)!N^i}$ is an exponentially small quantity. On the other hand, $B(N) \leq N^2 C(N)$, from which we conclude that $B(N)$ is exponentially small since the product of an exponentially small quantity and any polynomial in N remains an exponentially small quantity [13, Exercice 4.10, p. 158].

Therefore,

$$A(N) = \sum_{i=1}^{i_0} \frac{i^2 N!}{(N-i)! N^i} + \Delta(N),$$

where $\Delta(N)$ represents a function which is exponentially small.

For any integer i that is $o(N^{2/3})$ it is shown in [13, Proposition 4.4] that

$$\frac{N!}{(N-i)!N^i} = e^{-i^2/(2N)} \left(1 + \mathcal{O}\left(\frac{i}{N}\right) + \mathcal{O}\left(\frac{i^3}{N^2}\right)\right). \tag{18}$$

Since $i = o(N^{2/3})$ whenever $1 \le i \le i_0$, we deduce from (18) that

$$A(N) = \sum_{i=1}^{i_0} i^2 e^{-i^2/(2N)} \left(1 + \mathcal{O}\left(\frac{i}{N}\right) + \mathcal{O}\left(\frac{i^3}{N^2}\right)\right) + \Delta(N).$$

By applying the Euler-MacLaurin summation [13, Proposition 4.2] to the functions $x^3 e^{-x^2/2}$ and $x^5 e^{-x^2/2}$ we find that (see [13, Exercice 4.9] for similar results)

$$\sum_{i=1}^{i_0} i^2 e^{-i^2/(2N)} \mathcal{O}\left(\frac{i}{N}\right) = \mathcal{O}(N)$$

and

$$\sum_{i=1}^{i_0} i^2 e^{-i^2/(2N)} \mathcal{O}\left(\frac{i^3}{N^2}\right) = \mathcal{O}(N),$$

respectively. Hence,

$$A(N) = \sum_{i=1}^{i_0} i^2 e^{-i^2/(2N)} + \mathcal{O}(N). \tag{19}$$

By noting that $i^2 e^{-i^2/(2N)}$ is exponentially small for $i > i_0$ we can add all terms for $i > i_0$ into the r.h.s. of (19), which gives

$$A(N) = \sum_{i \ge 1} i^2 e^{-i^2/(2N)} + \mathcal{O}(N). \tag{20}$$

The above summation is the summation of the function $N x^2 e^{-x^2/2}$ at regularly spaced points with step $1/\sqrt{N}$. Another application of the Euler-MacLaurin formula [13, Proposition 4.2] yields

$$\sum_{i\geq 1} i^2 e^{-i^2/(2N)} = N^{3/2}\int_0^\infty x^2 e^{-x^2/2}dx + \mathcal{O}(N)$$
$$= N^{3/2}\sqrt{\frac{\pi}{2}} + \mathcal{O}(N), \quad (21)$$

so that

$$A(N) = N^{3/2}\sqrt{\frac{\pi}{2}} + \mathcal{O}(N)$$

from (20) and (21), which concludes the proof of the lemma.

■

References

[1] N. Bansal and Z. Liu. Capacity, delay and mobility in wireless ad-hoc networks. In *Proc. of IEEE Infocom 2003*, San Fransisco, CA, April 1-3 2003.

[2] C. Bettstetter. Mobility modeling in wireless networks: Categorization, smooth movement, and border effects. *ACM Mobile Computing and Communications Review*, 5(3):55–67, July 2001.

[3] C. Bettstetter and C. Wagner. The spatial node distribution of the random waypoint mobility model. *Proc. First German Workshop Mobile on Ad-Hoc Networks (WMAN 2002)*, pages 41–58, March 25-26 2002.

[4] J. Broch, D. Maltz, D. Johnson, Y.-C. Hu, and J. Jetcheva. Multi-hop wireless ad hoc network routing protocols. In *Proc. of ACM MobiCom '98*, pages 85–97, Dallas, TX, October 25-30 1998.

[5] W. Feller. *An Introduction to Probability Theory and its Applications.* John Wiley & Sons, Inc., 1950.

[6] R. Groenevelt, E. Altman, and P. Nain. Relaying in mobile ad hoc networks. In *Proc. of Workshop on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt'04)*, UK, March 2004.

[7] R. Guérin. Channel occupancy time distribution in a cellular radio system. *IEEE Transactions on Vehicular Technology*, 36:89–99, August 1987.

[8] D. B. Johnson and D. A. Maltz. Dynamic source routing in ad hoc wireless networks. In Imielinski and Korth, editors, *Mobile Computing*, volume 353. Kluwer Academic Publishers, 1996.

[9] Q. Li and D. Rus. Sending messages to mobile users in disconnected ad-hoc wireless networks. In *Proc. 6th ACM MobiCom*, Boston, MA, August 6-11 2000.

[10] D. Nain, N. Petigara, and H. Bakakrishnan. Integrated routing and storage for messaging applications in mobile ad hoc networks. In *Proc. of Workshop on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt'03)*, Sophia Antipolis, France, March 2003. To appear in MONET.

[11] P. Nain, D. Towsley, B. Liu, and Z. Liu. Properties of random direction models. In *Proc. of IEEE Infocom 2005*, Miami, FL, March 13-17 2005.

[12] W. Navidi and T. Camp. Stationary distributions for the random waypoint mobility model. *IEEE Transactions of Mobile Computing*, 3(1):99–108, January-March 2004.

[13] R. Sedgewick and P. Flajolet. *An Introduction to the Analysis of Algorithms*. Addison-Wiley, 1996.

[14] G. Sharma and R. R. Mazumdar. Delay and capacity trade-off in wireless ad hoc networks with random mobility. Submitted for publication, June 2004.

[15] G. Sharma and R. R. Mazumdar. Scaling laws for capacity and delay in wireless ad hoc networks with random mobility. In *Proc. of International Conference on Communications (ICC 2004)*, Paris, France, June 20-24 2004.

[16] J. Yoon, M. Liu, and B. Noble. Random waypoint considered harmful. In *Proc. of IEEE Infocom 2003*, pages 1312–1321, San Fransisco, CA, April 1-3 2003.

[17] W. Zhao and M. Ammar. Message ferrying: Proactive routing in highly-partioned wireless ad hoc networks. In *Proc. of the 9th IEEE Workshop on Future Trends in Distributed Computed Systems (FTDCS'03)*, San Juan, Puerto Rico, May, 28-30 2003.

Unité de recherche INRIA Sophia Antipolis
2004, route des Lucioles - BP 93 - 06902 Sophia Antipolis Cedex (France)

Unité de recherche INRIA Futurs : Parc Club Orsay Université - ZAC des Vignes
4, rue Jacques Monod - 91893 ORSAY Cedex (France)
Unité de recherche INRIA Lorraine : LORIA, Technopôle de Nancy-Brabois - Campus scientifique
615, rue du Jardin Botanique - BP 101 - 54602 Villers-lès-Nancy Cedex (France)
Unité de recherche INRIA Rennes : IRISA, Campus universitaire de Beaulieu - 35042 Rennes Cedex (France)
Unité de recherche INRIA Rhône-Alpes : 655, avenue de l'Europe - 38334 Montbonnot Saint-Ismier (France)
Unité de recherche INRIA Rocquencourt : Domaine de Voluceau - Rocquencourt - BP 105 - 78153 Le Chesnay Cedex (France)

Éditeur
INRIA - Domaine de Voluceau - Rocquencourt, BP 105 - 78153 Le Chesnay Cedex (France)
http://www.inria.fr
ISSN 0249-6399